Twisting Lightlike Solutions of the Kerr-Schild Class

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Abstract

Using a complex representation of the Debney–Kerr–Schild (DKS) solutions and the Kerr theorem we give a method to construct boosted Kerr geometries. In the ultrarelativistic case this method yelds twisting solutions having, contrary to the known pp-wave limiting solutions, a non-zero value of the total angular momentum. The solutions show that twist plays a crucial role in removing singularity and smoothing shock wave in the ultrarelativistic limit. Two different physical situations are discussed.

The problem of boosting of the black hole solutions received considerable attention in connection with different problems, like interaction between black holes and estimation of the gravitational interparticle interaction at very high energies.

First results in this field were obtained by Aichelburg and Sexl [1], who considered the boosting of the Schwarzschild solution. Due to the singular character of Lorentz transformations at $v = c$, many difficulties appear when the ultrarelativistic limit is involved. This singularity can, *a priori*, lead to different limiting results depending on the performed limiting procedure. Nevertheless, investigations of many authors led to a general conclusion that

black hole solutions turn, in the ultrarelativistic limits, into singular pp-waves (for references see [5]).

In the case of the boosting of the rotating Kerr BH, the pp-wave ultra-relativistic solution looses one of the main properties of the original solution, namely twist of the principal null congruence. In the same time, it is known that the Kerr-Schild class contains twisting lightlike solutions. The aim of this work is to show that there exist twisting lightlike solutions which can be considered as ultrarelativistic limits of the Kerr BH.

The approach used here is based on the DKS formalism [2] and on the Kerr theorem. It gives a possibility of obtaining exact and explicit expressions for the boosted Kerr geometry by arbitrary values and orientations of the boost with respect to the angular momentum.

The general Kerr-Schild metric is $g_{\mu\nu} = \eta_{\mu\nu} + 2he^3_\mu e^3_\nu$, where h is a scalar function and e^3 the principal null direction, given in null coordinates $(\zeta, \bar\zeta, u, v)$ by $e^3 = du + \bar Y d\zeta + Y d\bar\zeta - Y\bar Y dv$, where $Y(x)$ is a complex function. The Kerr theorem gives a rule to construct all geodesic, shear free congruences: the general geodesic, shear-free null congruence in Minkowski space is defined by a function Y which is a solution of the equation

$$F(\lambda_1, \lambda_2, Y) = 0 \ , \tag{1}$$

where F is an arbitrary analytic function of the *projective twistor coordinates* $\lambda_1 = \zeta - Yv, \qquad \lambda_2 = u + Y\bar\zeta, \qquad Y$. The quantity $\tilde r := -dF/dY$ is a complex radial distance, and singularities of the metric can be defined as the caustics of the congruence given by the system of equations

$$F = 0, \qquad dF/dY = 0 \ . \tag{2}$$

The Kerr solution belongs to the sub–class of solutions having singularities contained in a bounded region. In this case the function F must be at most quadratic in Y. The solutions of the equations (8) can be found in this case in explicit form, and correspond to the Kerr solution up to a Lorentz boost and a shift of the origin. Newman [3] constructed a complex representation which allows to represent the Kerr solution as a retarded-time field, generated by a complex source propagating along a complex world-line. In the complex version of the Kerr theorem [4] the function F depends on the coordinates of the complex world-line $x_0(\tau) = (\zeta_0, \bar\zeta_0, u_0, v_0) \in CM^4$. parameterized by a complex time parameter $\tau = t + i\sigma$.

The one-to-one correspondence between straight lines in complex Minkowski space and the class of the DKS solutions having singularities contained in a bounded region allows us to boost the Kerr solution *via* the DKS formalism. The general case of a solution with a boost corresponds to a straight, complex world line with 3-velocity $\vec{V}$ in CM^4

$$x_0^\mu(\tau) = x_0^\mu(0) + \xi^\mu \tau; \qquad \xi^\mu = (1, \vec{V}) \ . \tag{3}$$

It allows one to perform DKS-machinery and obtain solutions corresponding to parameters of the complex world line. On this way all the versions of the boosting of the Kerr BH can be considered in explicit form.

In particular, the most interesting case is that a boost collinear to the angular momentum. In this case, the Kerr singular ring grows as $a/\sqrt{1-v^2}$ and tends to infinity in the ultrarelativistic limit. In this limit, the function F acquires the form $F = x + iy - Y(z - ia - t)$. It follows that the complex radial distance is

$$\tilde{r} = -dF/dY = z - ia - t, \tag{4}$$

and therefore the metric has no singularity if a is non-zero. Two different physical situations must be considered in this case:

i) The field of a lightlike particle with a non-zero helicity $J = m_0 a_0 = ma$. The rest mass m_0 must be put equal to zero in the limit, so that $a_0 \to \infty$. The relativistic parameters m and a are kept constant, $h = m\frac{z-t}{(z-t)^2+a^2}$, so that the singularity on the front is absent.

ii) Relativistic boost of a particle with a finite rest mass m_0. In this case $m = m_0/\sqrt{1-v^2}$ grows under the effect of the boost, and consequently, $a = J/m = (J/m_0)\sqrt{1-v^2}$ is going to zero. It follows that h grows with the boost forming a wave with amplitude of order $\frac{m_0}{a_0(1-v^2)}$. However, m_0/a_0 is of order 10^{-44} for an electron, so that this effect can be observed only at $1 - v^2 \sim m_0/a_0 \sim 10^{-44}$.

In conclusion, in this approach twist plays a crucial role in removing singularity and smoothing shock wave in the ultrarelativistic limit.

References

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